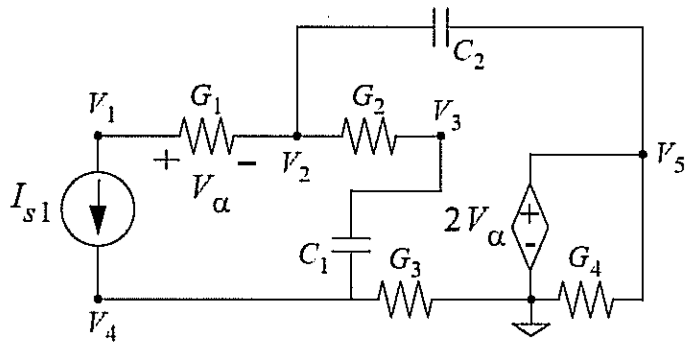
ECE4390 Lab 2 Quiz

Determine the node voltages for the circuit below in the frequency-domain at 20kHz. This question may only be done using MATLAB, your MNA program, by hand or any combination of the above. Full marks will be given for a correct final answer. If your answer is incorrect then part-marks will be awarded for any relevant work shown.



$G_1 = 10S$, $G_2 = 20S$, $G_3 = 10S$, $G_4 = 20S$, $C_1 = 0.5F$, $C_2 = 0.6F$, $I_{s1} = 1A$

$(sS+T)\bar{v} = \bar{w}$, $s = j\omega$, $\bar{v} = [V_1, V_2, V_3, V_4, V_5, I_1]^T$

From Ideal Element MNA Table:

$$S = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & C_2 & 0 & 0 & -C_2 & 0 \\ 0 & 0 & C_1 & -C_1 & 0 & 0 \\ 0 & 0 & -C_1 & C_1 & 0 & 0 \\ 0 & -C_2 & 0 & 0 & C_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad T = \begin{bmatrix} G_1 & -G_1 & 0 & 0 & 0 & 0 \\ -G_1 & G_1+G_2 & -G_2 & 0 & 0 & 0 \\ 0 & -G_2 & G_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & G_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & G_4 & 1 \\ -2 & 2 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad \bar{w} = \begin{bmatrix} -I_{s1} \\ 0 \\ 0 \\ I_{s1} \\ 0 \\ 0 \end{bmatrix}$$

Plug-in values, Solve Using Matlab and get:

$V_1 = -0.3V \angle 0^\circ$

$V_2 = -0.2V \angle 0^\circ$

$V_3 = -0.1V \angle 0^\circ$

$V_4 = -0.1V \angle 0^\circ$

$V_5 = -0.2V \angle 0^\circ$

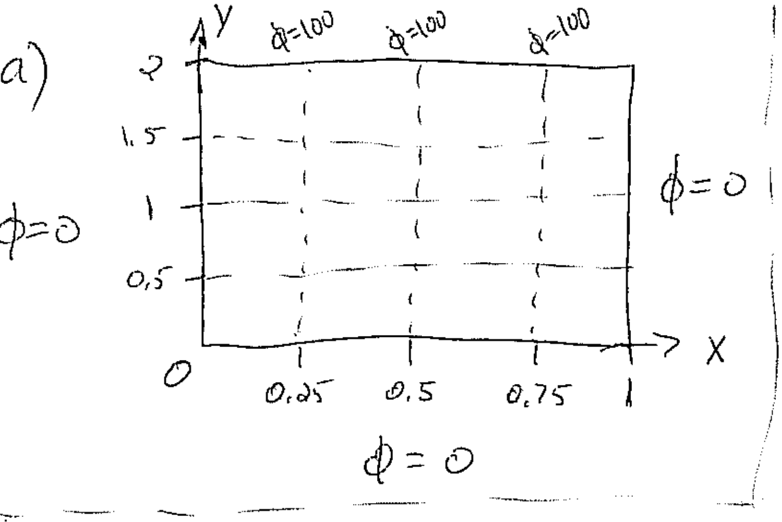
ECE4390 Lab 3 Quiz

For Laplace's equation $\nabla^2\phi(x,y) = 0$ over a rectangular domain with pure Dirichlet boundary conditions and a finite difference discretization such that $\Delta y = 2\Delta x$:

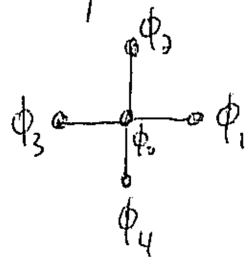
a) Draw this domain with a finite difference grid for $x = 0...1$ and $y = 0...2$, $\Delta x = 0.25$ and with $\phi(\xi, 2) = 100$ for $\xi = 0.25...0.75$ and $\phi = 0$ everywhere else on the boundary. Label any axes depicted.

b) Derive the SOR update equation for regular points (as opposed to interior points bounded by a Dirichlet boundary) for the problem described above. Use a central difference approximation for the derivative of ϕ. Note: the central difference formula can be taken directly from class notes and does not need to be derived. Hint: try deriving the Gauss-Seidel update equation first then proceed to the SOR equation.

a)



b) Computational Molecule:



$2\Delta x = \Delta y = h$,

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0$$

$\rightarrow$ Central Difference Approx:
$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} \approx \frac{\phi_1 - 2\phi_0 + \phi_3}{(h/2)^2} + \frac{\phi_2 - 2\phi_0 + \phi_4}{h^2}$$

$$\Rightarrow \frac{4(\phi_1 - 2\phi_0 + \phi_3)}{h^2} + \frac{\phi_2 - 2\phi_0 + \phi_4}{h^2} = 0 \quad \Leftrightarrow 4\phi_1 - 8\phi_0 + 4\phi_3 + \phi_2 - 2\phi_0 + \phi_4 = 0$$

$$\Leftrightarrow \phi_0 = \frac{4\phi_1 + 4\phi_3 + \phi_2 + \phi_4}{10} = \phi_{GS}$$ is Gauss-Seidel update scheme.

$\therefore$ SOR:

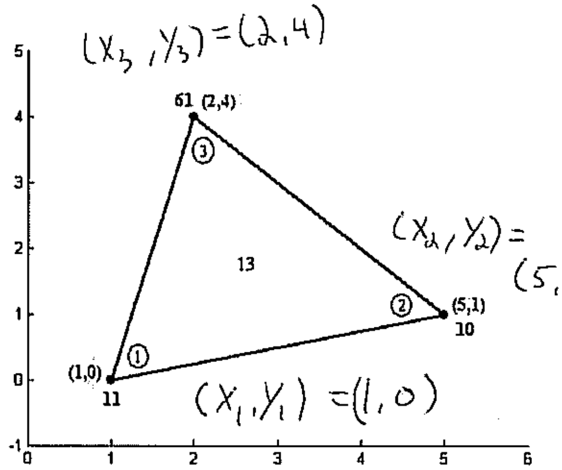
$$\phi_0^{n+1} = \phi_0^n + \omega(\phi_{GS} - \phi_0^n), \quad 1 < \omega < 2.$$

ECE4390 Lab 4 Quiz

Suppose that the solution to a 2D Poisson equation ($\nabla^2\Phi(x,y) = f(x,y)$) has been computed using the Finite Element Method (FEM) over a particular domain. As part of post-processing, determine the FEM-approximated values for $\phi(1,0)$, $\phi(2,4)$, $\phi(3,2)$, $\nabla\phi(4,2)$ and $\nabla\phi(3,1)$.

The information below has been extracted from the FEM program so that the above values can be found. The triangle is element number 13 from the domain mesh. Its global node numbers are in boldface, its local node numbers are circled, its node coordinates are in red and it's area is $7.5cm^2$. The FEM solution values at global nodes 10, 11 and 61 are also provided.

$$\phi(x,y) = \phi^{(e)}(x,y) = \underline{\phi}^T\underline{\alpha} = \underline{\alpha}^T\underline{\phi},$$



$\phi_{10} = 30$

$\phi_{11} = 15$

$\phi_{61} = 60$

$$\underline{\phi} = \begin{bmatrix} \phi_{①} \\ \phi_{②} \\ \phi_{③} \end{bmatrix} = \begin{bmatrix} \phi_{11} \\ \phi_{10} \\ \phi_{61} \end{bmatrix}, \quad \underline{\alpha} = \begin{bmatrix} \alpha_1(x,y) \\ \alpha_2(x,y) \\ \alpha_3(x,y) \end{bmatrix}$$

$$\alpha_i = \frac{1}{2A_e}(a_i + b_i x + c_i y), \quad a_i = (x_j y_k - x_k y_j), \quad b_i = (y_j - y_k), \quad c_i = (x_k - x_j)$$

$$\left.\begin{matrix} \text{for } i = 1,2,3 \\ j = 2,3,1 \\ k = 3,1,2 \end{matrix}\right\} \rightarrow$$

$$\Rightarrow \alpha_1 = \frac{1}{2(7.5)}\left[(20-3) + (1-4)x + (2-5)y\right] = \frac{1}{15}(18 - 3x - 3y)$$

$$\alpha_2 = \frac{1}{2(7.5)}\left[(0-4) + (4-0)x + (1-2)y\right] = \frac{1}{15}(-4 + 4x - y)$$

$$\alpha_3 = \frac{1}{2(7.5)}\left[(1-0) + (0-1)x + (5-1)y\right] = \frac{1}{15}(1 - x + 4y)$$

$$\text{So: } \phi(1,0) = \phi^{(e)}(1,0) = \phi_{11} = \boxed{15}, \quad \phi(2,4) = \phi^{(e)}(2,4) = \phi_{61} = \boxed{60},$$

$$\phi(3,2) = \phi^{(e)}(3,2) = \underline{\phi}^T\underline{\alpha}(3,2) = 15\left[\tfrac{1}{15}(18 - 3(3) - 3(2))\right] + 30\left[\tfrac{1}{15}(-4 + 4(3) - (2))\right]$$

$$+ 60\left[\tfrac{1}{15}(1 - (3) + 4(2))\right] = \boxed{39}$$

$$\nabla\phi(x,y) = \nabla\phi^{(e)}(x,y) = \nabla\underline{\phi}^T\underline{\alpha}(x,y) = \underline{\phi}^T\nabla\underline{\alpha}(x,y) = \underline{\phi}^T\begin{bmatrix} \frac{1}{2A_e}(b_1\hat{a}_x + c_1\hat{a}_y) \\ \frac{1}{2A_e}(b_2\hat{a}_x + c_2\hat{a}_y) \\ \frac{1}{2A_e}(b_3\hat{a}_x + c_3\hat{a}_y) \end{bmatrix}$$

$$\Rightarrow \nabla\phi(4,2) = \nabla\phi(3,1)$$

$$= 15\left[\tfrac{1}{15}(-3\hat{a}_x - 3\hat{a}_y)\right] + 30\left[\tfrac{1}{15}(4\hat{a}_x - \hat{a}_y)\right] + 60\left[\tfrac{1}{15}(-\hat{a}_x + 4\hat{a}_y)\right] = \boxed{\hat{a}_x + 11\hat{a}_y}$$

ECE4390 Lab 5 Quiz

Part I
A plane wave is traveling in free space in the positive y-direction with polarization in the positive x-direction. Assuming a right-handed coordinate system, determine a) the direction of the incident electric field of this wave in Cartesian coordinates; b) the velocity vector of the incident wave.

Part II
Given a 2-dimensional FDTD computational domain with $x \in [0, L_1]$ and $y \in [0, L_2]$ give the first order Mur absorbing boundary condition operators at boundaries $x = 0$, $x = L_1$, $y = 0$ and $y = L_2$.

Part I: From Class Notes on Transient plane Waves:

$$\hat{a}_r = -\hat{a}_y, \quad \Theta = \frac{\pi}{2}, \quad \phi = \frac{-\pi}{2}, \quad \zeta = \frac{\pi}{2}, \quad v_0 = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \simeq 3\times10^8 \text{ m/s}$$

a) $$\hat{e} = \left[\cos\frac{\pi}{2}\cos\frac{-\pi}{2}\cos\frac{\pi}{2} - \sin\frac{-\pi}{2}\sin\frac{\pi}{2},\ \cos\frac{\pi}{2}\sin\frac{-\pi}{2}\cos\frac{\pi}{2} + \cos\frac{\pi}{2}\sin\frac{\pi}{2},\ -\sin\frac{\pi}{2}\cos\frac{\pi}{2}\right.$$

$$\hat{e} = [1, 0, 0]\,[\hat{a}_x, \hat{a}_y, \hat{a}_z]^T$$

b) $$\underline{v} = v_0\left[-\sin\frac{\pi}{2}\cos\frac{-\pi}{2},\ -\sin\frac{\pi}{2}\sin\frac{-\pi}{2},\ -\cos\frac{\pi}{2}\right]$$

$$\underline{v} = v_0[0, 1, 0]\,[\hat{a}_x, \hat{a}_y, \hat{a}_z]^T$$

Part II: